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Technical Note

A study of heat transfer effectiveness of circular tubes with internal longitudinal fins having tapered lateral profiles

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Abstract

This paper presents a finite-difference based numerical simulation of steady, laminar heat transfer in circular tubes fitted with four identical longitudinal fins having tapered lateral profiles. The lateral view of the tip of each fin is a circular arc. The tube is exposed to constant heat flux. The temperature dependence of thermal conductivity and viscosity has been taken into account. The flow is assumed to be locally fully developed but thermally undeveloped. The momentum equation for the fluid and energy equations for the fluid and the tube wall with/without fins are solved iteratively and simultaneously. At each axial location, bulk velocity, bulk temperature of the fluid and Effectiveness which is an indicator of the enhancement of heat transfer due to addition of fins are calculated. A parametric study of effectiveness vs. axial distance for various combinations of fin materials and coolants reveals interesting results. The velocity profiles, friction factors and comparisons with respect to constant property solutions have also been discussed.

1. Introduction

1.1. General remarks

This paper presents a numerical simulation and optimization study of heat transfer in circular tubes fitted with identical internal longitudinal fins having tapered lateral profiles. Reynolds numbers less than or equal to 1500 (that is, much below the transition limit) are used in this study. Such problems find applications in the compact heat exchanger field where the coolant velocity must be reduced in order to lower the noisiness of the devices, to avoid excessive power dissipations or to prevent miniaturized structures from large pressure gradients.

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1.2. Literature review

Schmidt [12] suggested the adoption of a parabolic shape as an optimal profile for longitudinal fins. Such a proposition was supported by Duffin [3] on the basis of a rigorous variational model. Bejan and Morega [1] reports the optimal geometry of an array of fins that minimizes the thermal resistance between the substrate and the flow forced through the fins. The flow regime is laminar. The finite-difference results of Zhang and Faghri [14] show that adding internal fins is an efficient way to enhance heat transfer in thermal energy storage systems when a fluid with a low thermal conductivity is used as a transfer fluid. Olson [10] has measured heat transfer and pressure drop of three thin, compact heat exchangers in helium gas at 3.5 MPa and higher, with Reynolds number of 450–36 000. The measurements reveal that the heat exchanger with the pin fin internal geometry has significantly better heat transfer than the other geometries, but it also has higher pressure drop. The conjugate heat transfer in a high-performance finned-tube heat exchanger has been computed for three-dimensional thermally and hydrodynamically developing laminar flows by Fiebig et al. [6]. The influence

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Nomenclature

A	flow cross-sectional area (m^2)
c_p	specific heat of the coolant at constant pressure (J/kg K)
D_i	inner diameter of the tube (m)
D_o	outer diameter of the tube (m)
D_h	hydraulic diameter of the tube (m) $= 4A/P$
f	friction factor ($= -(dp/dz)/(\rho U_b^2/2D_i)$)
h	heat transfer coefficient (W/m^2 K)
k	thermal conductivity (W/m K)
L	length of the tube (m)
l_1	lateral length of the fin (m)
P	wetted perimeter (m)
p	pressure (N/m^2)
q''	heat flux (W/m^2)
Re	Reynolds number ($= \rho U_b D_h/\mu_m$)
r	radial coordinate (m), Fig. 1
r_i	inner radius of the tube (m)
r_o	outer radius of the tube (m)
T	temperature (K)
T_c	temperature of the coolant (K)
T_f	temperature of the fins or tube wall (K)
T_b	bulk or mixed mean temperature of the coolant (K)
T_{max}	maximum temperature on the outer surface of the tube (K)
U_b	bulk or mean velocity of the coolant (m/s)
u	axial velocity of the coolant (m/s)
z	axial coordinate (m), Fig. 1
Δz	axial step size (m)

Greek symbols

α	half-included angle of each fin (Fig. 2)
θ	circumferential coordinate (deg), Fig. 1
μ	coefficient of viscosity (N s/m^2)
μ_m	coefficient of viscosity at mean coolant temperature (N s/m^2)
ρ	density of the coolant (kg/m^3)

of Reynolds number and a fin efficiency parameter (ratio of fin to fluid conductivity times fin thickness to fin pitch) on the heat transfer behavior has been studied. Jang and Chen [9] have presented a numerical analysis of heat transfer and fluid flow in a three-dimensional wavy-fin and tube heat exchanger. Wen and Hsieh [13] have reported the results of experiments for saturated flow boiling of R-114, R-22, and R-134a in a water-heated, horizontal heat exchanger with internally spirally knurled/or integral finned tubes.

Queipo et al. [11] have used genetic algorithm in an electronics cooling problem where it is required to find optimal or nearly optimal arrangements of convectively cooled components placed in-line on the bottom wall of a ventilated two-dimensional channel. Fabbri [4] has proposed a genetic algorithm in order to optimize the thermal performance of finned surfaces. In a subsequent work, Fabbri [5] has studied the problem of optimizing the geometry of internally finned tubes in order to enhance the heat transfer under laminar flow conditions. The velocity and temperature distributions on the finned-tube cross-section are determined with the help of a finite element model.

2. Objectives of the present study

The objectives of the present study are: (a) to obtain a more rigorous solution as compared to that of Fabbri [5] by considering temperature-dependent properties such as viscosity and thermal conductivity; (b) to use a new fin geometry which is easy to manufacture by extrusion such as longitudinal fins having tapered lateral profiles, the lateral view of the tip of each fin being a circular arc; (c) to compute the enhancement of heat transfer in the internally finned tube subjected to constant heat flux under laminar flow conditions; (d) to carry out a parametric study of heat transfer effectiveness vs. axial distance for various combinations of fin materials and coolants.

2.1. Problem formulation

The problem is basically divided into two parts: (a) unfinned tube and (b) finned tube. The enhancement of heat transfer is evaluated in terms of *Effectiveness* which is defined as the ratio of heat transfer coefficient of the finned tube to that of the corresponding unfinned tube. The effectiveness is also a function of the axial location in the tube.

For the part (a), that is, for the unfinned tube, at each axial location the axial velocity of the fluid and the temperature of the tube wall and the fluid are functions of the radial coordinate only as the problem is axisymmetric. For the part (b), that is, for the finned tube the aforesaid variables (which now include fin temperature) are functions of both radial as well as circumferential coordinates due to inclusion of internal fins. It may be noted that the finned tube has four identical fins and axially symmetric cross-section. The tube has an internal diameter of 2 cm and thickness of 1 mm. Reynolds number of the flow(calculated based on the concept of hydraulic diameter) is less than or equal to 1500. The inlet temperature of the fluid is 25 °C. A uniform heat flux is imposed on the outer surface of the tube. The flow is assumed to be locally fully developed but thermally undeveloped. The coolants used are water and engine oil. The fin materials (same as the tube materials) used are aluminium, copper and carbon steel. The viscosity of the fluid and the thermal conductivities of the fin (and tube) and the fluid are functions of temperature. The relevant equations have been obtained from the data of Holman [8] by using curve fitting technique but not shown in this paper for the sake of brevity. The fin materials and the coolants have constant density and specific heat. The viscous dissipation and natural convection in the fluid have been neglected.

The front view and the sectional side view of the finned tube is shown in Fig. 1. The details of the geometry of one of the four identical fins is shown in Fig. 2. The lateral length of the fin (l_1) is 5 mm and the included half-angle (α) of the fin is 18°. The governing differential equations and the boundary conditions are as follows.

z-Momentum equation for the coolant:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{\partial u}{\partial r}\right)+\frac{1}{r}\frac{\partial}{\partial \theta}\left(\mu\frac{1}{r}\frac{\partial u}{\partial \theta}\right)=\frac{\mathrm{d}p}{\mathrm{d}z}, \tag{1}$$

where $\mu=\mu(T)$.

The boundary conditions are:

$$\text{at } r=0, \quad u=\text{finite}, \tag{2}$$

$$\text{at } r=r_{\mathrm{i}}, \quad \text{at the fin surfaces and at the fin tips,} \quad u=0. \tag{3}$$

Energy equation for the coolant:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_{\mathrm{c}}r\frac{\partial T_{\mathrm{c}}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_{\mathrm{c}}\frac{\partial T_{\mathrm{c}}}{\partial \theta}\right)=\rho c_{\mathrm{p}}u\frac{\partial T_{\mathrm{c}}}{\partial z}, \tag{4}$$

where $k_{\mathrm{c}}=k_{\mathrm{c}}(T)$.

Energy equation for the tube wall with fins:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_{\mathrm{f}}r\frac{\partial T_{\mathrm{f}}}{\partial r}\right)+\frac{1}{r^2}\frac{\partial}{\partial \theta}\left(k_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial \theta}\right)=0, \tag{5}$$

where $k_{\mathrm{f}}=k_{\mathrm{f}}(T)$.

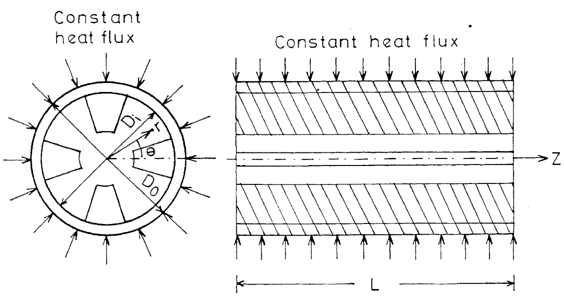

Fig. 1. The front view and the sectional side view of the finned tube.

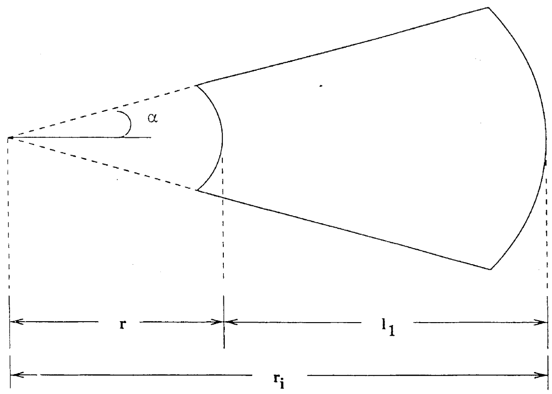

Fig. 2. The geometrical details of one of the four identical fins.

The boundary conditions are:

$$\text{at } r=0, \quad T_{\mathrm{c}}=\text{finite}, \tag{6}$$

$$\text{at } r=r_{\mathrm{i}}, \quad k_{\mathrm{c}}\frac{\partial T_{\mathrm{c}}}{\partial r}=k_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial r}, \tag{7}$$

$$T_{\mathrm{c}}=T_{\mathrm{f}} \tag{8}$$

at the fin surfaces,

$$\frac{k_{\mathrm{c}}}{r}\frac{\partial T_{\mathrm{c}}}{\partial \theta}=\frac{k_{\mathrm{f}}}{r}\frac{\partial T_{\mathrm{f}}}{\partial \theta}, \tag{9}$$

$$T_{\mathrm{c}}=T_{\mathrm{f}} \tag{10}$$

at the fin tips,

$$k_{\mathrm{c}}\frac{\partial T_{\mathrm{c}}}{\partial r}=k_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial r}, \tag{11}$$

$$T_{\mathrm{c}}=T_{\mathrm{f}}, \tag{12}$$

$$\text{at } r=r_0, \quad k_{\mathrm{f}}\frac{\partial T_{\mathrm{f}}}{\partial r}=q'' \tag{13}$$

2.2. Heat transfer coefficient (h)

$$h=\frac{q''}{T_{\max}-T_{\mathrm{b}}}. \tag{14}$$

2.3. Effectiveness

At any axial location of the tube, *Effectiveness* is calculated by evaluating the ratio of the heat transfer coefficient of the finned tube to that of the unfinned tube.

3. Method of solution

The governing equations are discretized using finite-difference method. For the unfinned tube, the condition at the center is handled by using L'Hospital's rule, whereas for the finned tube, the center condition is handled by taking a very small square region around the center so that governing differential equations in Cartesian coordinates are valid in that region [7]. The interfaces between the fluid and the fins as well as that between the fluid and the tube wall are taken care of by deriving a finite difference equation for the interface grid points by satisfying energy equations for the solid and the fluid and the compatibility conditions using a Taylor series expansion for temperatures in the immediate neighbourhood of the interface grid points [2].

The computation starts with guessed temperature fields in the fluid and the solid. At any axial station, the momentum equation for the fluid and the energy equations for the fluid and the tube wall with/without fins are solved iteratively and simultaneously. For the unfinned tube, tri-diagonal matrix algorithm (TDMA) is used to solve the set of simultaneous equations at each axial location. For the finned tube, Gauss–Seidel iterative method is applied. A pure implicit scheme is used to march in the axial direction from the entrance to the exit.

Grid independence tests are performed for both cases. The optimal number of grid points in the computational domain of the unfinned tube (including the tube wall) is 25 while that for the finned tube (including the fins and the tube wall) is 25×20, the first and second number indicating the number of grid points in r-direction and θ-direction, respectively. The number of grid points in the tube wall is four including the interface between the wall and the fluid. The grid spacings in the radial direction in the fluid are different from those in the fins and the tube wall although they are uniform in each region. Δz is taken as 0.1 m.

The accuracy of the solution is checked against the limiting case solution ($Nu = 4.364$) for laminar flow at very large axial location(that is, far from the tube entrance) of the unfinned tube of internal diameter of 2 cm with thin wall (having a thickness of 0.00025 m) and constant properties both for the fluid (engine oil) and the tube (made of carbon steel) subjected to a constant heat flux of $q'' = 1$ KW/m^2. The pressure gradient ($\mathrm{d}p/\mathrm{d}z$) is -2×10^5 N/m^3. The result is found to be very satisfactory.

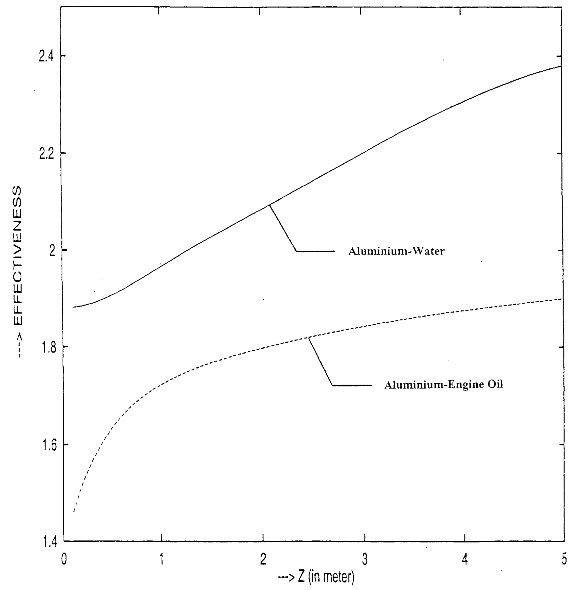

Fig. 3. Effectiveness vs. z plot for aluminium–water and aluminium–engine oil combinations for $q'' = 10$ kW/m^2.

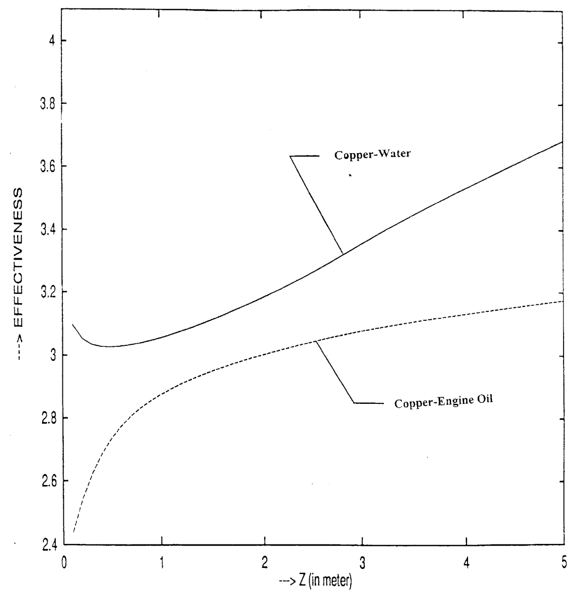

Fig. 4. Effectiveness vs. z plots for copper–water and copper–engine oil combinations for $q'' = 10$ kW/m^2.

4. Results and discussion

4.1. Effectiveness vs. axial distance

Figs. 3–5 show the effectiveness vs. axial distance plots for aluminium–water and aluminium–engine oil, copper–water and copper–engine oil, and carbon steel–water and carbon steel–engine oil for $q'' = 10$ KW/m^2, respectively. For water, $\mathrm{d}p/\mathrm{d}z = -15$ N/m^3 and for engine oil, $\mathrm{d}p/\mathrm{d}z = -2 \times 10^5$ N/m^3. It is seen that (a) at every z-station, effectiveness is greater than one for any combination of fin material and the fluid. This means there is heat transfer enhancement due to addition of fins. (b) For all three cases water is a more

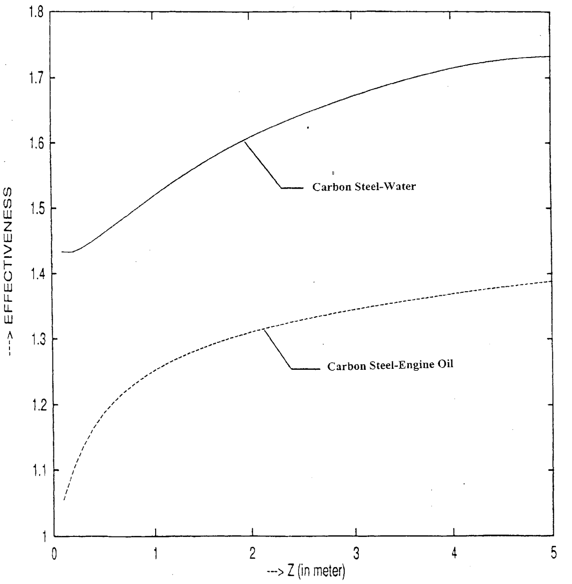

Fig. 5. Effectiveness vs. z plots for carbon steel–water and carbon steel–engine oil combinations for $q'' = 10\ \mathrm{kW/m^2}$.

effective coolant. This is because of higher thermal conductivity of water as compared to engine oil which results in lower temperature difference between the tube and the fluid thus producing higher heat transfer coefficient and hence, less resistance to heat transfer. (c) In all three cases, with oil as a coolant, effectiveness increases with axial distance, effectiveness being maximum with copper as the fin material and minimum with carbon steel as the fin material. The reason for the rise of effectiveness with axial distance is that with axial distance the temperature of the tube wall and fins increase which results in better heat transfer effectiveness. As for the other observation, copper has the largest thermal conductivity among the three fin materials considered and hence copper as the fin material gives rise to best effectiveness. (d) An interesting phenomenon is noticed when water is used as a coolant. The effectiveness falls a little near the entrance of the tube and then rises again. This is most pronounced when copper is the fin material. This may be due to the fact that rate of rise of tube wall and fin temperature with axial distance is much lower near the entrance when water is used as a coolant because water has a higher thermal conductivity as compared to engine oil. Since the conductivity ratio for copper–water is highest as compared to aluminium–water and carbon steel–water, the drop of the effectiveness is most visible in the case of copper–water combination. Away from the entrance, the temperature rise of the tube wall and the fin is significant enough to produce an increase in effectiveness.

4.2. Velocity profiles in the finned tube

Fig. 6 shows the axial fluid velocity profiles for the aluminium–water combination with respect to the circumferential location in the tube at a radial distance, $r = 0.0045$ m (that is, the 10th grid point away the center) and at three axial locations in the tube, namely, $z = 0.1$ m, $z = 2$ m, and $z = 5$ m. Since the 11th grid point in the radial direction coincides with the fin tips, the aforesaid velocity distributions are indicative of the flow characteristics just before the fin tips. Each tic on the x-axis is 18° apart from each other and corresponds to the circumferential location of a grid point on the $r = 0.0045$ m circle. The velocity values at the grid points are joined by straight line segments. The plots clearly reveal the periodic and symmetric nature of the flow. The lower velocities are near the fin tips and higher velocities are in the region between the successive fins. $\theta = 0°$ (and 360)°, 90°, 180°, 270° lines are the fin tip center lines, θ being measured anti-clockwise (see Fig. 1). Furthermore, as the axial distance increases, the flow velocity also increases because of the drop in viscosity with the rise of temperature as the fluid flows from the inlet to the exit.

It may also be noted that till $r = 0.001$ m, no effect of the fins on the velocity profile is observed. That is, there is no circumferential variation of flow velocity very near the center of the finned tube.

4.3. Influence of variable properties on effectiveness

Calculations based on both constant and variable properties indicate that while there is an insignificant change in effectiveness for water, the value of effectiveness rises to an appreciable extent for engine oil for all fin materials when constant properties (that is, mean viscosity and conductivity values) are used. For exam-

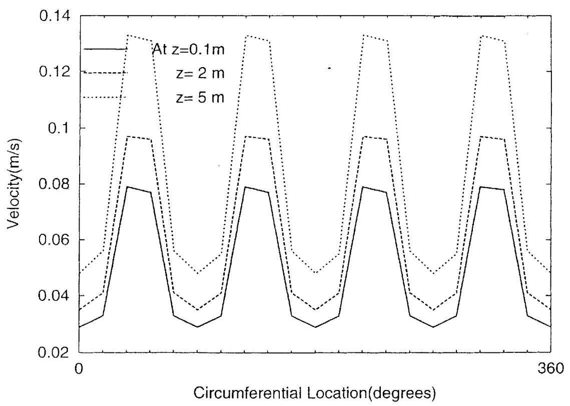

Fig. 6. Axial velocities with respect to circumferential locations in the finned tube at $r = 0.0045$ m at various axial distances for aluminium–water combination.

ple, at $z = 1$ m, for copper–engine oil combination, there is a rise of 25.8% in effectiveness when constant properties are used. The corresponding value for aluminium–water combination is 1.84%. The reason is that viscosity of engine oil is a much stronger function of temperature as compared to water.

4.4. Effect of fins on friction factor

As expected, addition of fins enhances the friction factors. For example, at $z = 1$ m, for aluminium–water combination, the values of f are 0.187 and 0.0588 for finned and unfinned tubes, respectively. The corresponding values for copper–engine oil combination are 8.884 and 0.623. The higher values for oil as compared to water may be attributed to much greater viscosity of the former. It may also be noted that the present numerically computed friction factor for the limiting case of isothermal (298 K), laminar flow in an unfinned tube matches very closely with the corresponding analytical solution.

5. Conclusions

This work shows a numerical simulation of steady, laminar heat transfer in internally finned circular tubes subjected to constant heat flux. The geometry of the fins is new and has not been investigated earlier. The results show significance enhancement of heat transfer due to the internal fins. Water turns out to be a more effective coolant as compared to engine oil. Although the general trend is found to be increase of effectiveness along the length of the tube, interestingly, with water as a coolant, for all fin materials, the effectiveness drops a little near the tube entrance (this phenomenon is most pronounced in case of copper fins) and then rises again with the axial distance in the rest of the tube. The velocity profiles, friction factors and comparisons with respect to constant property solutions have also been discussed.

References

[1] A. Bejan, A.M. Morega, Optimal arrays of pin fins and plate fins in laminar forced convection, ASME J. Heat Transfer 115 (1993) 75–81.

[2] B. Carnahan, H.A. Luther, J.O. Wilkes, Applied Numerical Methods, Wiley, New York, 1969.

[3] R.J. Duffin, A variational problem related to cooling fins, J. Math. Mech. 8 (1959) 47–56.

[4] G. Fabbri, A genetic algorithm for fin profile optimization, Int. J. Heat Mass Transfer 40 (9) (1997) 2165–2172.

[5] G. Fabbri, Heat transfer optimization in internally finned tubes under laminar flow conditions, Int. J. Heat Mass Transfer 41 (10) (1998) 1243–1253.

[6] M. Fiebig, A. Grosse-Gorgemann, Y. Chen, N.K. Mitra, Conjugate heat transfer of a finned tube, Part A: heat transfer behaviour and occurrence of heat transfer reversal, Numer. Heat Transfer, Part A 28 (1995) 133–146.

[7] P.S. Ghoshdastidar, Computer Simulation of Flow and Heat Transfer, Tata McGraw-Hill Publishing Company, New Delhi, 1998.

[8] J.P. Holman, Heat Transfer, eighth ed., McGraw-Hill, New York, 1997.

[9] J.-Y. Jang, L.-K. Chen, Numerical analysis of heat transfer and fluid flow in a three-dimensional wavy-fin and tube heat exchanger, Int. J. Heat Mass Transfer 40 (16) (1997) 3981–3990.

[10] D.A. Olson, Heat transfer in thin, compact heat exchangers with circular, rectangular, or pin–fin flow passages, ASME J. Heat Transfer 114 (1992) 373–382.

[11] N. Queipo, R. Devarakonda, J.A.C. Humphrey, Genetic algorithms for thermosciences research: application to the optimized cooling of electronic components, Int. J. Heat Mass Transfer 37 (6) (1994) 893–908.

[12] E. Schmidt, Die warmeuebertragung durch rippen, Zeitschrift des Vereines Deutscher Ingenieure 70 (1926) 885–951.

[13] M.-Y. Wen, S.-S. Hsieh, Saturated flow boiling heat transfer in internally spirally knurled/integral finned tubes, ASME J. Heat Transfer 117 (1995) 245–248.

[14] Y. Zhang, A. Faghri, Heat transfer enhancement in latent heat thermal energy storage system by using the internally finned tube, Int. J. Heat Mass Transfer 39 (15) (1996) 3165–3173.